Packet Switching in Radio Channels: Part I—Carrier Sense Multiple-Access Modes and Their Throughput-Delay Characteristics

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Abstract—Radio communication is considered as a method for providing remote terminal access to computers. Digital byte streams from each terminal are partitioned into packets (blocks) and transmitted in a burst mode over a shared radio channel. When many terminals operate in this fashion, transmissions may conflict with and destroy each other. A means for controlling this is for the terminal to sense the presence of other transmissions; this leads to a new method for multiplexing in a packet radio environment: carrier sense multiple access (CSMA). Two protocols are described for CSMA and their throughput-delay characteristics are given. These results show the large advantage CSMA provides as compared to the random ALOHA access modes.

I. INTRODUCTION

LARGE COMPUTER installations, enormous data banks, and extensive national computer networks are now becoming available. They constitute large expensive resources which must be utilized in a cost/effective fashion. The constantly growing number of computer applications and their diversity render the problem of *accessing* these large resources a rather fundamental one. Prior to 1970, wire connections were the principal means for communication among computers and between users and computers. The reasons were simple: dial-up and leased telephone lines were available and could provide inexpensive and reasonably reliable communications for short distances, using a readily available and widespread technology. It was long recognized that this technology was inadequate for the needs of a computer-communication system which is required to handle bursty traffic (i.e., large peak to average data rates). For example, the inadequacies included the long dial-up and connect time, the minimum three-minute tariff structure, the fixed and limited data rates, etc. However, it was not until 1969 that the cost to switch communication bandwidth dropped below the cost of the bandwidth being switched [1]. At that time, the new technology of packet-switched computer networks emerged and developed a cost/effective means for connecting computers together over long-distance high-speed lines. However, these networks did not solve the *local* interconnection problem, namely, how can one efficiently provide access from the user to the network itself? Certainly, one solution is to use wire connections here also. An alternate solution is the subject of this paper, namely, ground radio packet switching.

Paper approved by the Associate Editor for Computer Communication of the IEEE Communications Society for publication after presentation at the National Computer Conference, Anaheim, Calif., 1975. Manuscript received December 5, 1974; revised June 11, 1975. This work was supported in part by the Advanced Research Projects Agency, Department of Defense under Contract DAHC15-73-C-0368.

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We wish to consider broadcast radio communications as an alternative for computer and user communications. The ALOHA System [2] appears to have been the first such system to employ wireless communications. The advantages in using broadcast radio communications are many: easy access to central computer installations and computer networks; collection and dissemination of data over large distributed geographical areas independent of the availability of preexisting (telephone) wire networks; the suitability of wireless connections for communications with and among mobile users (a constantly growing area of interest and applications); easily bypassed hostile terrain; etc. Perhaps, this broadcast property is the key feature in radio communication.

The Advanced Research Projects Agency (ARPA) of the Department of Defense recently undertook a new effort whose goal is to develop new techniques for packet radio communication among geographically distributed, fixed or mobile, user terminals and to provide improved frequency management strategies to meet the critical shortage of RF spectrum. The research presented in this paper is an integral part of the total design effort of this system which encompasses many other research topics [3]–[9].

Consider an environment consisting of a number of (possibly mobile) users in line-of-sight and within range of each other, all communicating over a (broadcast) radio channel in a common frequency band. The classical approach for satisfying the requirement of two users who need to communicate is to provide a communication channel for their use so long as their need continues (line-switching). However, the measurements of Jackson and Stubbs [10] show that such allocation of scarce communication resources is extremely wasteful. Rather than providing channels on a user-pair basis, we much prefer to provide a single high-speed channel to a large number of users which can be shared in some fashion. This, then, allows us to take advantage of the powerful "large number laws" which state that with very high probability, the demand at any instant will be approximately equal to the sum of the average demands of that population. We wish to take advantage of these gains due to resource sharing.

Of interest to this paper is the consideration of radio channels for packet switching (also called packet radio channels). A packet is merely a package of data prepared by one user for transmission to some other user in the system. As soon as we deal with shared channels in a packet-switching mode, then we must be prepared to resolve conflicts which arise when more than one demand is simultaneously placed upon the channel. In packet radio channels, whenever a portion of one user's transmission overlaps with another user's transmission, the two collide and "destroy" each other. The existence of some acknowledgment scheme permits the transmitter to determine if his transmission was successful or not. The problem we are faced with is how to control the access to the channel in a fashion which produces, under the physical constraints of simplicity and hardware implementation, an acceptable level of performance. The difficulty in controlling a channel which must carry its own control information gives rise to the so-called random-access modes. A simple scheme, known as "pure ALOHA," permits users to transmit any time they desire. If, within some appropriate time-out period, they receive an acknowledgment from the destination, then they know that no conflicts occurred. Otherwise, they assume a collision occurred and they must retransmit. To avoid continuously repeated conflicts, some scheme must be devised for introducing a *random* retransmission delay, spreading the conflicting packets over time. A second method for using the radio channel is to modify the completely unsynchronized use of the ALOHA channel by "slotting" time into segments whose duration is exactly equal to the transmission time of a single packet (assuming constant-length packets). If we require each user to start his packets only at the beginning of a slot, then when two packets conflict, they will overlap completely rather than partially, providing an increase in channel efficiency. This method is referred to as "slotted ALOHA" [11]–[13].

The radio channel as considered in this paper is characterized as a wide-band channel with a propagation delay between any source-destination pair which is very small compared to the packet transmission time.[1] This suggests a third approach for using the channel; namely, the carrier sense multiple-access (CSMA) mode. In this scheme one attempts to avoid collisions by listening to (i.e., "sensing") the carrier due to another user's transmission.[2] Based on this information about the state of the channel, one may think of various actions to be taken by the terminal. Two protocols will be described and analyzed which we call "persistent" CSMA protocols: the nonpersistent and the p-persistent CSMA. Below, we present the protocols, discuss the assumptions, and finally establish and display the throughput-delay performance for each.

[1] Consider, for example, 1000-bit packets transmitted over a channel operating at a speed of 100 kbits/s. The transmission time of a packet is then 10 ms. If the maximum distance between the source and the destination is 10 mi, then the (speed of light) packet propagation delay is of the order of 54 μs. Thus the propagation delay constitutes only a very small fraction ($a = 0.005$) of the transmission time of a packet. On the contrary, when one considers satellite channels [13] the propagation delay is a relatively large multiple of the packet transmission time ($a \gg 1$).

[2] Sensing carrier prior to transmission is a well-known concept in use for (voice) aircraft communication. In the context of packet radio channels, it was originally suggested by D.Wax of the University of Hawaii in an internal memorandum dated Mar. 4, 1971.

II. CSMA TRANSMISSION PROTOCOLS AND SYSTEM ASSUMPTIONS

The various protocols considered below differ by the action (pertaining to packet transmission) that a terminal takes after sensing[3] the channel. However, in all cases, when a terminal learns that its transmission was unsuccessful, it reschedules the transmission of the packet according to a randomly distributed retransmission delay. At this new point in time, the transmitter senses the channel and repeats the algorithm dictated by the protocol. At any instant a terminal is called a *ready terminal* if it has a packet ready for transmission at this instant (either a new packet just generated or a previously conflicted packet rescheduled for transmission at this instant).

A terminal may, at any one time, either be transmitting or receiving (but not both simultaneously). However, the delay incurred to switch from one mode to the other is negligible. Furthermore, the time required to detect the carrier due to packet transmissions is negligible (that is a zero detection time is assumed).[4] All packets are of constant length and are transmitted over an assumed noiseless channel (i.e., the errors in packet reception caused by random noise are not considered to be a serious problem and are neglected in comparison with errors caused by overlap interference). The system assumes noncapture (i.e., the overlap of any fraction of two packets results in destructive interference and both packets must be retransmitted). We further simplify the problem by assuming the propagation delay (small compared to the packet transmission time) to be identical[5] for all source–destination pairs.

We first consider the *nonpersistent CSMA*. The idea here is to limit the interference among packets by always rescheduling a packet which finds the channel busy upon arrival. More precisely, a ready terminal senses the channel and operates as follows.

1) If the channel is sensed idle, it transmits the packet.

2) If the channel is sensed busy, then the terminal schedules the retransmission of the packet to some later time according to the retransmission delay distribution. At this new point in time, it senses the channel and repeats the algorithm described.

A slotted version of the nonpersistent CSMA can be

[3] Each terminal has the capability of sensing carrier on the channel. The practical problems of feasibility and implementation of sensing, however, are not addressed here.

[4] The detection time is considered negligible for relatively wide-band channels (100 kHz). In Part II [19] the detection time on the "busy-tone" narrow-band channels (on the order of 2 kHz) will be accounted for in the analysis.

[5] By considering this constant propagation delay equal to the largest possible, one gets lower (i.e., pessimistic) bounds on performance.

considered in which the time axis is slotted and the slot size is τ seconds (the propagation delay). All terminals are synchronized[6] and are forced to start transmission only at the beginning of a slot. When a packet's arrival occurs during a slot, the terminal senses the channel at the beginning of the next slot and operates according to the protocol described above.

We next consider the *p-persistent CSMA* protocol. However, before treating the general case (arbitrary p), we introduce the special case of $p = 1$.

The *1-persistent CSMA* protocol is devised in order to (presumably) achieve acceptable throughput by never letting the channel go idle if some ready terminal is available. More precisely, a ready terminal senses the channel and operates as follows.

1) If the channel is sensed idle, it transmits the packet with probability one.

2) If the channel is sensed busy, it waits until the channel goes idle (i.e., persisting on transmitting) and only then transmits the packet (with probability one—hence, the name of 1-persistent).

A slotted version of this 1-persistent CSMA can also be considered by slotting the time axis and synchronizing the transmission of packets in much the same way as for the previous protocol.

The above 1-persistent and nonpersistent protocols differ by the probability (one or zero) of not rescheduling a packet which upon arrival finds the channel busy. In the case of a 1-persistent CSMA, we note that whenever two or more terminals become ready during a transmission period (TP), they wait for the channel to become idle (at the end of that transmission) and then they all transmit with probability one. A conflict will also occur with probability one! The idea of randomizing the starting time of transmission of packets accumulating at the end of a TP suggests itself for interference reduction and throughput improvement. The scheme consists of including an additional parameter p, the probability that a ready packet persists ($1 - p$ being the probability of delaying transmission by τ seconds). The parameter p will be chosen so as to reduce the level of interference while keeping the idle periods between any two consecutive nonoverlapped transmissions as small as possible. This gives rise to the *p-persistent CSMA*, which is a generalization of the 1-persistent CSMA.

More precisely, the protocol consists of the following: the time axis is finely slotted where the (mini) slot size is τ seconds. For simplicity of analysis, we consider the system to be synchronized such that all packets begin their transmission at the beginning of a (mini) slot.

Consider a ready terminal. If the channel is sensed idle, then: with probability p, the terminal transmits the packet; or with probability $1 - p$, the terminal delays the transmission of the packet by τ seconds (i.e., one slot). If at this new point in time, the channel is still detected idle, the same process is repeated. Otherwise, some packet must have started transmission, and our terminal schedules the retransmission of the packet according to the retransmission delay distribution (i.e., acts as if it had conflicted and learned about the conflict).

If the ready terminal senses the channel busy, it waits until it becomes idle (at the end of the current transmission) and then operates as above.

III. TRAFFIC MODEL: ASSUMPTIONS AND NOTATION

In the previous section, we identified the system protocols, operating procedures, and assumptions. Here we characterize the traffic source and its underlying assumptions.

We assume that our traffic source consists of an infinite number of users who collectively form an independent Poisson source with an aggregate mean packet generation rate of λ packets/s. This is an approximation to a large but finite population in which each user generates packets infrequently and each packet can be successfully transmitted in a time interval much less than the average time between successive packets generated by a given user. Each user in the infinite population is assumed to have at most one packet requiring transmission at any time (including any previously blocked packet).

In addition, we characterize the traffic as follows. We have assumed that each packet is of constant length requiring T seconds for transmission. Let $S = \lambda T$. S is the average number of packets generated per transmission time, i.e., it is the input rate normalized with respect to T. Under steady-state conditions, S can also be referred to as the channel throughput rate. Now, if we were able to perfectly schedule the packets into the available channel space with absolutely no overlap or gaps between the packets, we could achieve a maximum throughput equal to 1; therefore we also refer to S as the *channel utilization*. Because of the interference problem inherent in the random nature of the access modes, the achievable throughput will always be less than 1. The maximum achievable throughput for an access mode is called the *capacity* of the channel under that mode.

Since conflicts can occur, some acknowledgment scheme is necessary to inform the transmitter of its success or failure. We assume a positive acknowledgment scheme[7]: if within some specified delay (an appropriate time-out period) after the transmission of a packet, a user does not receive an acknowledgment, he knows he has conflicted. If he now retransmits immediately, and if all users behave likewise, then he will definitely be interfered with again (and forever!). Consequently, as mentioned above, each user delays the transmission of a previously collided packet by some random time whose mean is $\bar{X}$ (chosen, for example, uniformly between 0 and $X_{\max} = 2\bar{X}$). The traffic

[6] In this paper, the practical problems involved in synchronizing terminals are not addressed.

[7] The channel for acknowledgment is assumed to be separate from the channel we are studying (i.e., acknowledgments arrive reliably and at no cost).

offered to the channel from our collection of users consists not only of new packets but also of previously collided packets: this increases the mean *offered* traffic rate which we denote by G (packets per transmission time T) where $G \geq S$.

Our two further assumptions are the following.

Assumption 1: The average retransmission delay $\bar{X}$ is large compared to T.

Assumption 2: The interarrival times of the point process defined by the start times of all the packets plus retransmissions are independent and exponentially distributed.

It is clear that Assumption 2 is violated in the protocols we consider. (We have introduced it for analytic simplicity.) However, in Section V, some simulation results are discussed which show that performance results based on this assumption are excellent approximations, particularly when the average retransmission delay $\bar{X}$ is large compared to T. Moreover, in the context of slotted ALOHA it was analytically shown [14] in the limit as $\bar{X} \rightarrow \infty$, that Assumption 2 is satisfied; furthermore, simulation results showed that only the first moment of the retransmission delay distribution had a noticeable effect on the average throughput-delay performance.

So far, we have defined the following important system variables: S (throughput), G (offered channel traffic rate), T (packet transmission time), $\bar{X}$ (average retransmission delay), τ (propagation delay), and p (p-persistent parameter). Without loss of generality, we choose $T = 1$. This is equivalent to expressing time in units of T. We express $\bar{X}$ and τ in these normalized time units as $\delta = \bar{X}/T$ and $a = \tau/T$.

IV. THROUGHPUT ANALYSIS

We wish to solve for the channel capacity of the system for all of the access protocols described above. This we do by solving for S in terms of G (as well as the other system parameters). The channel capacity is then found by maximizing S with respect to G. S/G is merely the probability of a successful transmission and G/S is the average number of times a packet must be transmitted (or scheduled) until success. In Section V, we discuss delay and give the throughput-delay tradeoff for these protocols.

This analysis is based on renewal theory and probabilistic arguments requiring independence of random variables provided by Assumption 2. Moreover steady-state conditions are assumed to exist. However from the (S,G) relationships found below one can see that steady state may not exist because of inherent instability of these random-access techniques. This instability is simply explained by the fact that when statistical fluctuations in G increase the level of mutual interference among transmissions, then the positive feedback causes the throughput to decrease to 0. Nevertheless, the results are useful for the following reasons.

1) They are meaningful for a finite (and possibly long) period of time. (Simulations supporting these analytic results showed no saturation over the simulated period of time when $\bar{X}$ was large enough; see Section V.)

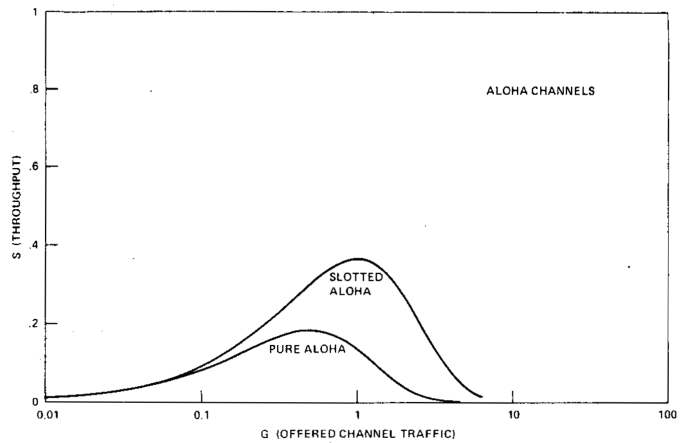

Fig. 1. Throughput in ALOHA channels.

2) In finite population cases, stable situations are possible for which steady-state results prevail over an infinite time horizon. (See [14] and [16].)

3) Control procedures have been prescribed for the slotted ALOHA random access [14] which stabilize unstable channels, achieving performance very close to the equilibrium results.

A. ALOHA Channels

In the *pure ALOHA* access mode, each terminal transmits its packet over the data channel in a completely unsynchronized manner. Under the system and model assumptions (mainly Assumption 2), we have

$$S = GP_s$$

where P_s is the probability that an arbitrary offered packet is successful. A given packet will overlap with another packet if there exists at least one start of transmission within T seconds before or after the start time of the given packet (i.e., over a "vulnerable" interval of length $2T$). Using the Poisson traffic assumption, Abramson [2] first showed that

$$S = Ge^{-2G}. \tag{1}$$

Thus, we see that pure ALOHA achieves a maximum throughput of $1/(2e) = 0.184$ (at $G = 1/2$).

In the *slotted ALOHA*, if two packets conflict, they will overlap completely rather than partially (i.e., a vulnerable interval only of length T). The throughput equation then becomes

$$S = Ge^{-G} \tag{2}$$

and was first obtained by Roberts [12] who extended Abramson's result in (1). With this simple change, the maximum throughput is increased by a factor of two to $1/e = 0.368$ (at $G = 1$). In Fig. 1, we plot the throughput S versus the offered traffic G for these two systems. From these results, it is all too evident that a significant fraction of the channel's ultimate capacity ($C = 1$) is not utilized with the ALOHA access modes; we recover a major portion of this loss with the CSMA protocols, as we now show.

B. Nonpersistent CSMA

The basic equation for the throughput S is expressed in terms of a (the ratio of propagation delay to packet transmission time) and G (the offered traffic rate) as follows:

$$S = \frac{Ge^{-aG}}{G(1 + 2a) + e^{-aG}}. \tag{3}$$

Proof: G denotes the arrival rate of new and rescheduled packets. All arrivals, in this case, do not necessarily result in actual transmissions (a packet which finds the channel in a busy state is rescheduled without being transmitted). Thus, G constitutes the "offered" channel traffic and only a fraction of it constitutes the channel traffic itself. Consider the time axis[8] (See Fig. 2)[9] and let t be the time of arrival of a packet which senses the channel idle and such that no other packet is in the process of transmission. Any other packet arriving between t and $t + a$ will find (sense) the channel as unused, will transmit, and hence will cause a conflict. If no other terminal transmits a packet during these a seconds (the "vulnerable" period), then the first packet will be successful.

Let $t + Y$ be the time of occurrence of the last packet arriving between t and $t + a$. The transmission of all packets arriving in $(t, t + Y)$ will be completed at $t + Y + 1$. Only a seconds later will the channel be sensed unused. Now, any terminal becoming ready between $t + a$ and $t + Y + 1 + a$ will sense the channel busy and hence will reschedule its packet. The interval between t and $t + Y + 1 + a$ is called a *transmission period* (TP). Note that there can be at most one successul transmission during a TP. Define an *idle period* to be the period of time between two consecutive TP's (also called busy periods in this simple case). A busy period plus the following idle period constitute a cycle. Let $\bar{B}$ be the expected duration of the busy period, $\bar{I}$ the expected duration of the idle period, and $\bar{B} + \bar{I}$ the expected length of a cycle. Let U denote the time during a cycle that the channel is used without conflicts. Using renewal theory arguments, the average channel utilization is simply given by

$$S = \frac{\bar{U}}{\bar{B} + \bar{I}}. \tag{4}$$

The probability that a TP is successful is simply the probability that no terminal transmits during the first a seconds of the period and is equal to e^{-aG}. Therefore

$$\bar{U} = e^{-aG}. \tag{5}$$

The average duration of an idle period is simply $1/G$. The average duration of a busy interval is $1 + \bar{Y} + a$, where $\bar{Y}$ is the expected value of Y.

[8] The reference time axis considered in this and subsequent proofs is the transmitter's time. Shifting all transmissions by τ seconds will give a description of events on the station's time axis. Any time overlap in transmission on the station's time axis results in packet interference.

[9] In this and other figures, a vertical arrow represents a terminal becoming ready.

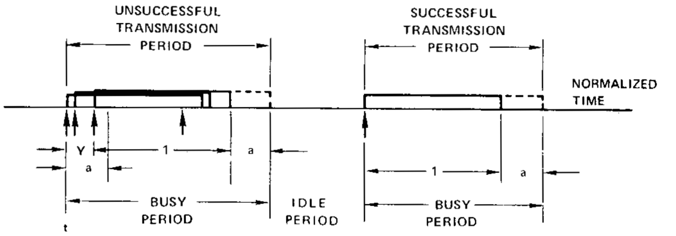

Fig. 2. Nonpersistent CSMA: Busy and idle periods.

The distribution function for Y is

$$F_Y(y) \triangleq \Pr\{Y \le y\} = \Pr\{\text{no arrival occurs in an interval of length } a - y\} = \exp\{-G(a - y)\}, \qquad (y \le a). \tag{6}$$

The average of Y is therefore given by

$$\bar{Y} = a - \frac{1}{G}(1 - e^{-aG}). \tag{7}$$

Applying (4) and using the expressions found for $\bar{U}$, $\bar{B}$, and $\bar{I}$, we get (3). Q.E.D.

It is easy to prove that the throughput equation for the *slotted* nonpersistent CSMA is given by

$$S = \frac{aGe^{-aG}}{(1 - e^{-aG}) + a}. \tag{8}$$

Note that for both cases we have

$$\lim_{a \to 0} S = G/(1 + G). \tag{9}$$

This shows that when $a = 0$, a throughput of 1 can theoretically be attained for an offered channel traffic equal to infinity. S versus G for various values of a is plotted in Fig. 3.

C. 1-Persistent CSMA

The throughput equation for this protocol is given by

$$S = \frac{G[1 + G + aG(1 + G + aG/2)]e^{-G(1+2a)}}{G(1 + 2a) - (1 - e^{-aG}) + (1 + aG)e^{-G(1+a)}}. \tag{10}$$

Proof: Consider Fig. 4 and again let t be the time of arrival of a packet which senses the channel to be idle with no other packet in the process of transmission. In this protocol, any packet arriving in the interval $[t + a, t + Y + 1 + a]$ will sense the channel busy and hence must wait until the channel is sensed idle (at time $t + 1 + Y + a$) at which time they will *all* transmit! The number of packets accumulated at the end of TP is the number of arrivals in $1 + Y$ seconds. If this total is equal to or greater than two, then a conflict occurs in the next TP with probability 1.

Define a *busy period* to be the time between t and the end of that TP during which no packets accumulate. De-

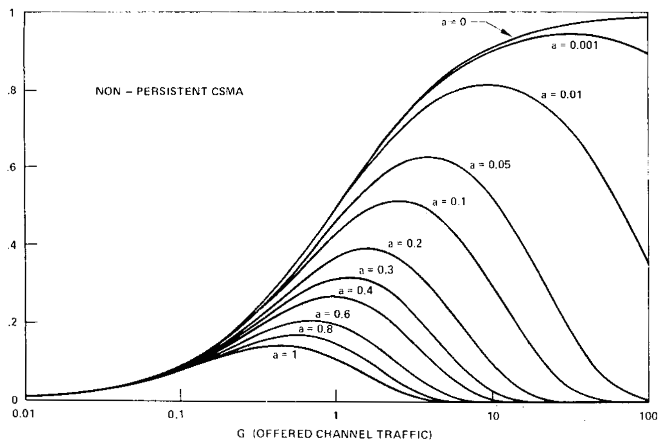

Fig. 3. Throughput in nonpersistent CSMA.

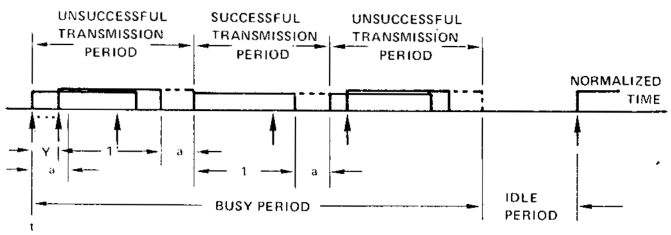

Fig. 4. 1-persistent CSMA: TP's, busy periods, and idle periods.

fine an *idle period* to be the period of time in which the channel is idle and no packets are present awaiting transmission. A busy period plus the following idle period constitute a *cycle*.

Let $\bar{B}$ be the expected duration of the busy period, $\bar{I}$ the expected duration of the idle period, and $\bar{B} + \bar{I}$ the expected length of a cycle.

Let us now consider the transmission of an arbitrary packet. Three situations must be considered.

1) If the packet arrives to an idle system, then its transmission is successful if and only if no packets arrive during its first a seconds; its probability of success is therefore e^{-aG}.

2) If the packet arrives during the first a seconds of a TP, then its probability of success is 0.

3) If the packet arrives during the channel busy period (excluding the first a seconds of the TP), then it is successful (in the next TP) if and only if it is the only packet to arrive during this TP and no packets arrive during *its* first a seconds. To calculate its probability of success, we observe that a TP is of random length equal to $1 + a + Y$ where Y is a random variable. Let B' denote the time during a cycle that the channel is in its busy period excluding the first a seconds of each TP. B' is a sequence of segments of random length $1 + Y \triangleq Z$ separated by periods of a seconds. Knowing that a packet arrives in B', this packet is more likely to arrive in a longer segment Z than in a shorter one (due to the "paradox of residual life" [17]). Let $\hat{Z}$ denote the segment in which the arrival occurred, and $\hat{q}_0$ (derived below) be the probability that no arrival occurs in $\hat{Z}$; the probability of success of the packet is therefore $\hat{q}_0 e^{-aG}$.

Only cases 1) and 3) contribute to a successful transmission. Let $\bar{B}'$ be the expected value of B'. From renewal theory arguments, the probability that an arrival finds the channel idle [case (1)] is given by $\bar{I}/(\bar{B} + \bar{I})$, and the probability that an arrival finds the channel in situation 3) is $\bar{B}'/(\bar{B} + \bar{I})$; then the probability of success of the packet is given by

$$P_s \triangleq \Pr\{\text{success}\} = \frac{\bar{I}}{\bar{B} + \bar{I}} e^{-aG} + \frac{\bar{B}'}{\bar{B} + \bar{I}} \hat{q}_0 e^{-aG}. \quad (11)$$

The determination of $\bar{I}, \bar{B}, \bar{B}'$, and $\hat{q}_0$ follows.

Since the traffic is Poisson, it is clear that the average idle period is given by

$$\bar{I} = 1/G. \quad (12)$$

For $\bar{B}, \bar{B}'$, and $\hat{q}_0$ we must first obtain some intermediate results as follows. The distribution function for Y and its average are given in (6) and (7), respectively. The Laplace transform of the probability density function of Y, defined as

$$F_Y^*(s) \triangleq \int_0^\infty e^{-sy}\, dF_Y(y),$$

is given by

$$F_Y^*(s) = e^{-aG} + \frac{G(e^{-as} - e^{-aG})}{G - s}. \quad (13)$$

Let us now find the distribution of the number of packets accumulated at the end of a TP.

Let

$$q_m(y) \triangleq \Pr\{m \text{ packets accumulated at end of TP} \mid Y = y\}$$

and

$$q_m = \int_0^a q_m(y)\, dF_Y(y).$$

Let $Q(z)$ denote the generating function of q_m defined by

$$Q(z) \triangleq \sum_{m=0}^{\infty} q_m z^m.$$

The number of packets accumulated at the end of a TP is equal to the number of packets arriving during a period of time equal to $1 + Y$. Let m_1 denote the number of packets arriving in $T = 1$, and m_2 the number of packets arriving in Y. Let $Q_1(z)$ and $Q_2(z)$ denote the generating functions of the probability distributions for m_1 and m_2, respectively. Since the arrival process is Poisson, the random variables m_1 and m_2 are independent and the generating function $Q(z)$ of q_m, where $m = m_1 + m_2$, is given by

$$Q(z) = Q_1(z)Q_2(z).$$

We have [17]

$$Q_1(z) = \exp\{G(z - 1)\}$$

and

$$Q_2(z) = F_Y^*(G(1 - z)).$$

From (13) we get

$$Q(z) = \exp\{G(z-1)\}\exp\{-aG\}\left[1 + \frac{\exp\{aGz\} - 1}{z}\right]. \quad (14)$$

We may invert this explicit expression for $Q(z)$; in particular we find that the probability of zero packets accumulated at the end of a TP is

$$q_0 = Q(z)\,|_{z=0} = \exp\{-G(1+a)\}[1+aG]. \quad (15)$$

To find the average busy period, we let Y_i denote the random variable Y defined above corresponding to the ith TP in a busy period. All Y_i, $i = 1,2,\cdots$, are independent and identically distributed. It is easy to see that the number of TP's in a busy period is geometrically distributed with mean $1/q_0$. Conditioned on the fact that we have exactly k TP's in the busy period and that $Y_i = y_i$ for $i = 1,2,\cdots,k$, the average busy period is

$$\bar{B}(y_1,y_2,\cdots,y_k) = k(1+a) + y_1 + y_2 + \cdots + y_k.$$

Therefore, by removing the conditions on k and Y_i, we get $\bar{B}$ as

$$\bar{B} = \cdots\int_{y_i=0}^{a}\cdots\int_{y_1=0}^{a}\sum_{k=1}^{\infty}[k(1+a) + y_1 + \cdots + y_k] \cdot q_0(y_k)\prod_{i=1}^{k-1}(1-q_0(y_i))\,dF_{Y_1}(y_1)\cdots dF_{Y_i}(y_i)\cdots.$$

It is easy to see that by inverting the order of summation and integration, the contribution of the term $k(1+a)$ reduces to $(1+a)/q_0$ and the contribution of the generic term y_j simply reduces to $\bar{Y}(1-q_0)^{j-1}$. Finally, we have

$$\bar{B} = \frac{1+a}{q_0} + \sum_{j=1}^{\infty}\bar{Y}(1-q_0)^{j-1} = \frac{1+a+\bar{Y}}{q_0}. \quad (16)$$

Since the average number of TP's is $1/q_0$, from the distribution of B' we have

$$\bar{B}' = \frac{1+\bar{Y}}{q_0}. \quad (17)$$

In (11) for P_s, it remains only to compute $\hat{q}_0$. The probability density function of $Z = 1 + Y$ is easily obtained from the distribution of Y. From (6), the probability density function of Y can be expressed as

$$f_Y(y) = \exp\{-aG\}u_0(y) + G\exp\{-aG\}\exp\{Gy\}, \qquad 0 \le y \le a$$

where $u_0(y)$ is the unit impulse at $y = 0$. Thus we have

$$f_Z(x) = \exp\{-aG\}u_0(x-1) + G\exp\{-aG\} \cdot \exp\{G(x-1)\}, \qquad 1 \le x \le 1+a.$$

The probability density function of $\hat{Z}$ is given by [17]

$$f_{\hat{Z}}(x) = \frac{xf_Z(x)}{\bar{Z}} = \frac{e^{aG}}{1+\bar{Y}}u_0(x-1) + \frac{Gxe^{-aG}e^{G(x-1)}}{1+\bar{Y}}, \qquad 1 \le x \le 1+a.$$

Finally, the probability that no arrival occurs (from our Poisson source) in the interval $\hat{Z}$ is simply

$$\hat{q}_0 = \int_{x=1}^{1+a}\exp\{-Gx\}f_{\hat{Z}}(x)\,dx = \frac{\exp\{-G(1+a)\}}{1+\bar{Y}}[1+aG(1+a/2)]. \quad (18)$$

Using our expressions for $\bar{I}$, $\bar{B}$, $\bar{B}'$, and $\hat{q}_0$ in (12), (16), (17), and (18), respectively, we immediately obtain from (11)

$$P_s = \frac{\dfrac{1+\bar{Y}}{q_0}e^{-aG}\hat{q}_0 + \dfrac{1}{G}e^{-aG}}{\dfrac{1+a+\bar{Y}}{q_0} + \dfrac{1}{G}}.$$

Substituting the expressions obtained for q_0, $\hat{q}_0$, and $\bar{Y}$, and recalling that $S = GP_s$, we have finally established (10). Q.E.D.

Slotted 1-persistent CSMA: Let us now consider the *slotted* version of 1-persistent CSMA. The throughput equation for this case is given by

$$S = \frac{G\exp\{-G(1+a)\}[1+a-\exp\{-aG\}]}{(1+a)(1-\exp\{-aG\}) + a\exp\{-G(1+a)\}} \quad (19)$$

Proof: In this slotted version, as in slotted ALOHA, if two packets conflict, they will overlap completely. The length of a TP is always equal to $1 + a$. (We have assumed that the packet transmission time is an integer multiple of the propagation delay.)

Since the traffic process is an independent one (Assumption 2), the number of slots in an idle period is geometrically distributed with a mean equal to $1/(1 - e^{-aG})$. Thus the average idle period is given by

$$\bar{I} = \frac{a}{1-e^{-aG}}. \quad (20)$$

Using a similar argument, we find that the average busy period is given by

$$\bar{B} = \frac{1+a}{\exp\{-G(1+a)\}}. \quad (21)$$

Let $\bar{U}$ again denote the expected time during a cycle that the channel is used without conflicts. In order to find $\bar{U}$ we need to determine the probability of success over each

TP in the busy period. The probability of success over the first TP is given by

$$\Pr\{\text{success over first TP}\} = \Pr\{\text{only one packet arrives during the last slot of the preceding idle period/some arrival occurred}\} = \frac{aGe^{-aG}}{1-e^{-aG}}.$$

Similarly we have:

$$\Pr\{\text{success over any other TP}\} = \frac{G(1+a)\exp\{-G(1+a)\}}{1-\exp\{-G(1+a)\}}.$$

The number of TP's in a busy period is geometrically distributed with a mean equal to

$$\exp\{G(1+a) \triangleq 1/q_0,$$

thus

$$\bar{U} = \frac{aG\exp\{-aG\}}{1-\exp\{-aG\}} + \left(\frac{1}{q_0}-1\right) \cdot \frac{G(1+a)\exp\{-G(1+a)\}}{1-\exp\{-G(1+a)\}}. \tag{22}$$

Applying (4) and using the expressions found for $\bar{U}$, $\bar{I}$, and $\bar{B}$, we get (19). Q.E.D.

The ultimate performance in the ideal case ($a = 0$), for both slotted and unslotted versions, is

$$S = \frac{Ge^{-G}(1+G)}{G+e^{-G}}. \tag{23}$$

For any value of a, the maximum throughput S will occur at an optimum value of G. In Fig. 5 we show S versus G for the nonslotted version of 1-persistent CSMA for various values of a.

D. p-Persistent CSMA

For a given offered traffic G and a given value of the parameter p, we can determine the throughput S as

$$S(G,p,a) = \frac{(1-e^{-aG})[P_s'\pi_0 + P_s(1-\pi_0)]}{(1-e^{-aG})[a\bar{t}'\pi_0 + a\bar{t}(1-\pi_0)+1+a] + a\pi_0} \tag{24}$$

where P_s', P_s, $\bar{t}'$, $\bar{t}$, and π_0 are defined in the following proof in (37), (34), (36), (30), and (25), respectively.

Proof: Consider a TP and assume that some packets arrive during the period as shown in Fig. 6. These packets sense the channel busy and accumulate at the end of the TP, at which point they randomize the starting times of their transmission according to the randomizing process described in Section II. This randomization creates a random delay before a TP starts, called the initial random transmission delay (IRTD), during which time the channel is "wasted." If, at the start of a new TP, two or more terminals decide to transmit, then a conflict will certainly occur. All other packets which have delayed their transmission by τ seconds will then sense the channel busy and will have to be rescheduled for transmission by incurring a retransmission delay δ. Thus, at the expense of creating this IRTD, we greatly improve the probability of success over a TP.

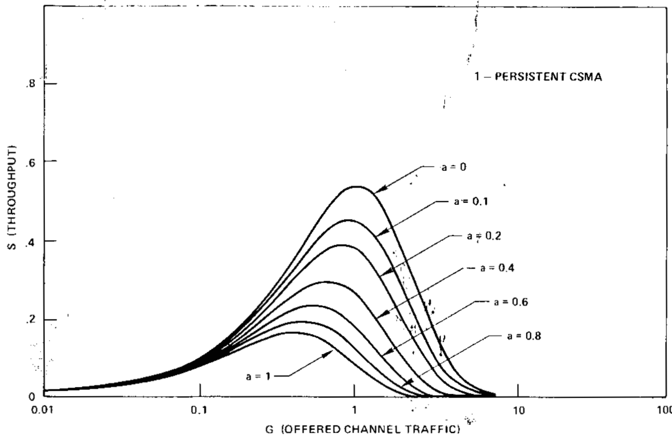

Fig. 5. Throughput in 1-persistent CSMA.

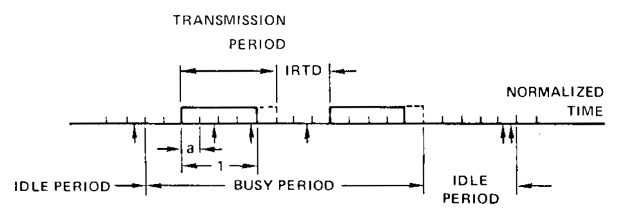

Fig. 6. *p*-persistent CSMA: TP's, busy periods, and idle periods.

Consider Fig. 6 in which we observe two TP's separated by an IRTD. One can also define busy periods and idle periods in much the same way as before. An idle period is that period of time during which the channel is idle and no packets are ready for transmission. A busy period consists of a sequence of transmission periods such that some packets arrive during each transmission period *except* the last one. Let $\mathfrak{J}_i$ denote the ith TP of a busy period. In order to find the channel utilization, we once again apply (4), which requires identifying and determining the average busy and idle periods, the gaps between TP's, as well as the condition for success over each TP. This we do as follows.

Recall that we require the system to be (mini-) slotted (the slot size equal to a, the normalized propagation delay) and all transmissions to start at the beginning of a slot. Here again we consider the transmission time of a packet to be an integer number $1/a$ slots (recall $T = 1$). Let $g = aG$; g is the average arrival rate of new and rescheduled packets during a (mini) slot.

We first determine the distribution of the number of packets accumulated at the end of a TP. Let N denote this number and let $\pi_n \triangleq \Pr\{N = n\}$. According to the protocol described in Section II, only those packets arriving during a TP will accumulate at the end of that TP. Therefore, by Assumption 1, we have

$$\pi_n = \frac{[(1+a)G]^n}{n!} \exp\{-(1+a)G\}, \qquad n \geq 0. \quad (25)$$

To find the distribution of the IRTD between two successive TP's in the same busy period, we condition $N = n$ and we let t_n be the number of slots elapsed until some packet is transmitted. Let $q = 1 - p$. It is easy to see that

$$\begin{aligned}\Pr\{t_n > k\} &= q^{(k+1)n} \prod_{j=1}^{k} \left[\sum_{m=0}^{\infty} \exp\{-g\} \frac{g^m}{m!} q^{m(k-j+1)} \right] \\ &= q^{(k+1)n} \prod_{j=1}^{k} \exp\{g(q^{k-j+1} - 1)\} \\ &= q^{(k+1)n} \exp\left\{ g\left(\frac{q(1-q^k)}{p} - k \right) \right\} \quad (26)\end{aligned}$$

and, therefore, for $k > 0$ we have

$$\begin{aligned}\Pr\{t_n = k\} &= \Pr\{t_n > k-1\} - \Pr\{t_n > k\} \\ &= q^{kn}[1 - q^n \exp\{-g(1-q^k)\}] \\ &\quad \cdot \exp\left\{ g\left(\frac{q(1-q^{k-1})}{p} - (k-1) \right) \right\} \quad (27)\end{aligned}$$

and for $k = 0$,

$$\Pr\{t_n = 0\} = 1 - q^n. \quad (28)$$

The average IRTD is given by

$$\bar{t}_n = \sum_{k=0}^{\infty} \Pr\{t_n > k\}. \quad (29)$$

Removing the condition on N, we get

$$\bar{t} = \sum_{n=1}^{\infty} \bar{t}_n \frac{\pi_n}{1 - \pi_0}. \quad (30)$$

$\bar{t}$ is the average gap between two consecutive TP's in a busy period.

In order to find the probability of success over a TP $\mathfrak{I}_i$ one has to distinguish two cases: $i = 1$ and $i \neq 1$. We first treat the second case, $i \neq 1$. Given $N = n$, define[10]:

- $P_s(n)$ probability of success over $\mathfrak{I}_i$
- L_n the number of packets present at the starting time of $\mathfrak{I}_i$
- $L_n - n$ merely the number of packets arriving during the gap t_n.

By the Poisson assumption we have

$$\Pr\{L_n = l / t_n = k\} = \frac{(kg)^{l-n}}{(l-n)!} e^{-kg}, \qquad l \geq n. \quad (31)$$

[10] The quantities $P_s(n)$, P_s, and L_n need no index i since they are identical for all $\mathfrak{I}_i$, $i \neq 1$.

Removing the condition on t_n,

$$\Pr\{L_n = l\} = \sum_{k=1}^{\infty} \frac{(kg)^{l-n}}{(l-n)!} e^{-kg} \Pr\{t_n = k\} + (1 - q^n)\delta_{l,n}, \qquad l \geq n \quad (32)$$

where $\delta_{i,j}$ is the Kronecker delta. The probability of success over $\mathfrak{I}_i$ is equal to the probability that none of the L_n transmit over $\mathfrak{I}_i$:

$$P_s(n) = \sum_{l=n}^{\infty} \frac{lpq^{l-1}}{1 - q^l} \Pr\{L_n = l\}. \quad (33)$$

Removing the condition on N, we get

$$P_s = \sum_{n=1}^{\infty} P_s(n) \frac{\pi_n}{1 - \pi_0}. \quad (34)$$

For the probability of success over $\mathfrak{I}_1$ we note that the number of packets present at the beginning of a busy period, denoted by N', is the number of packets arriving in the last slot of the previous idle period. We then have

$$\begin{aligned}\pi_n &\triangleq \Pr\{N' = n\} \\ &= \frac{g^n}{n!} \frac{e^{-g}}{1 - e^{-g}}, \qquad n \geq 1. \quad (35)\end{aligned}$$

Given $N' = n$, let t_n' denote the first initial random transmission delay of the busy period, and $P_s'(n)$ denote the probability of success over $\mathfrak{I}_1$. The distribution of t_n' and its average $\bar{t}_n'$ are the same as for t_n [(27) and (29)]. $P_s'(n)$ is the same as $P_s(n)$ [see (33)]. Removing the condition on N', we get

$$\bar{t}' = \sum_{n=1}^{\infty} t_n' \pi_n' \quad (36)$$

$$P_s' = \sum_{n=1}^{\infty} P_s'(n) \pi_n'. \quad (37)$$

It remains to compute $\bar{B}$, $\bar{U}$, and $\bar{I}$. It is clear that the number of TP's in a busy period is equal to m with probability $\pi_0(1 - \pi_0)^{m-1}$.

Consider a busy period with m TP's. Let N_i denote the number of packets accumulated at the end of the ith TP. We know that $N_m = 0$, and that all other $N_i \geq 1$ are independent and identically distributed random variables. Conditioned on the fact that $N_i = n_i, i = 1, \cdots, m-1$, the average busy period is given by

$$\bar{B}_m(n_1, \cdots, n_{m-1}) = a\bar{t}' + \sum_{i=1}^{m-1} a\bar{t}_{n_i} + m(1 + a). \quad (38)$$

The expected time, during the busy period, that the channel is used without conflicts is given by

$$\bar{U}_m(n_1, \cdots, n_{m-1}) = P_s' + \sum_{i=1}^{m-1} P_s(n_i). \quad (39)$$

On the other hand, we know that

$$\Pr\{N_i = n_i\} = \frac{\pi_{n_i}}{1-\pi_0}, \qquad n_i \geq 1,\ i = 1,2,\cdots,m-1. \tag{40}$$

Therefore, removing the conditions $N_i = n_i$ in (38) and (39), we get

$$\bar{B}_m = a\bar{t}' + (m-1)a\bar{t} + m(1+a) \tag{41}$$

$$\bar{U}_m = P_s' + (m-1)P_s \tag{42}$$

and removing the condition on m we get

$$\bar{B} = \sum_{m=1}^{\infty} \bar{B}_m \pi_0 (1-\pi_0)^{m-1} = a\bar{t}' + \frac{a\bar{t}(1-\pi_0)+1+a}{\pi_0} \tag{43}$$

$$\bar{U} = P_s' + \frac{1-\pi_0}{\pi_0} P_s. \tag{44}$$

The idle period is geometrically distributed with mean $1/(1-e^{-g})$; its average is:

$$\bar{I} = \frac{a}{1-e^{-g}}. \tag{45}$$

Finally, using (4) and substituting for $\bar{B}$, $\bar{U}$, and $\bar{I}$ the expressions found in (43), (44), and (45), respectively, we get the throughput S; it is a function of G, p, and $a = 1/T$ and is expressed as

$$S(G,p,a) = \frac{P_s' + \dfrac{1-\pi_0}{\pi_0} P_s}{a\bar{t}' + a\bar{t}\dfrac{1-\pi_0}{\pi_0} + \dfrac{1+a}{\pi_0} + \dfrac{a}{1-e^{-g}}} \tag{46}$$

which reduces to (24). Q.E.D.

In order to evaluate $S(G,p,a)$, a PL/1 program was written and run on the IBM 360/91 of the Campus Computing Network at UCLA. For small values of p $(0.01 \leq p \leq 0.1)$, the numerical computation as suggested by (24) becomes time consuming and requires an extremely large amount of storage. Fortunately some approximations have been found useful which lead to a closed-form solution for the throughput (see the derivations of $S'(G,p,a)$ in Appendix A).

Special case $a = 0$: Let us now consider the special case $a = 0$. For finite G, $g = aG = 0$. Equation (26) becomes

$$\Pr\{t_n > k\} = q^{(k+1)n}.$$

The average IRTD is then given by (29), and is expressed as

$$\bar{t}_n = \sum_{k=0}^{\infty} \Pr\{t_n > k\} = \frac{q^n}{1-q^n}.$$

It is important to note that $\bar{t}_n$ is finite, so is $\bar{t}$. On the other hand the idle period given in (45) becomes

$$\bar{I} = \frac{1}{G}.$$

Since $\bar{t}$ and $\bar{t}'$ are finite, by letting $a \to 0$ in (46) we get

$$S(G,p,a=0) = \frac{P_s' + \dfrac{1-\pi_0}{\pi_0} P_s}{\dfrac{1}{\pi_0} + \dfrac{1}{G}}. \tag{47}$$

To compute P_s we have to get back to (31) through (34). With $a = 0$ we have

$$\Pr\{L_n = l/t_n = k\} = 1, \qquad l = n$$

and

$$\Pr\{L_n = n\} = 1.$$

Therefore

$$P_s(n) = \frac{npq^{n-1}}{1-q^n} \tag{48}$$

and

$$P_s = \sum_{n=1}^{\infty} \frac{npq^{n-1}}{1-q^n} \frac{\pi_n}{1-\pi_0} \tag{49}$$

where

$$\pi_n = \frac{G^n}{n!} e^{-G}. \tag{50}$$

By the same token, we see from (35) that

$$\pi_1' = \frac{ge^{-g}}{1-e^{-g}} \underset{g \to 0}{\longrightarrow} 1$$

and that

$$P_s' = P_s'(1) = 1.$$

With these considerations, the throughput is given by

$$S(G,p,a=0) = \frac{G[\pi_0 + (1-\pi_0)P_s]}{G+\pi_0} \tag{51}$$

where P_s and π_n are given in (49) and (50), respectively. When $p = 1$, we have, from (48),

$$P_s(1) = 1$$

$$P_s(n) = 0, \qquad n > 1$$

and therefore

$$P_s = \frac{Ge^{-G}}{1-e^{-G}}.$$

Equation (51) then becomes

$$S(G, p=1, a=0) = \frac{G(1+G)e^{-G}}{G+e^{-G}}$$

which is (and should be) identical to the 1-persistent CSMA when $a = 0$ [see (23)]. Let us now consider $p \to 0$. Since $1 - q_n \approx np$, (48) then becomes

$$P_s(n) = q^{n-1}$$

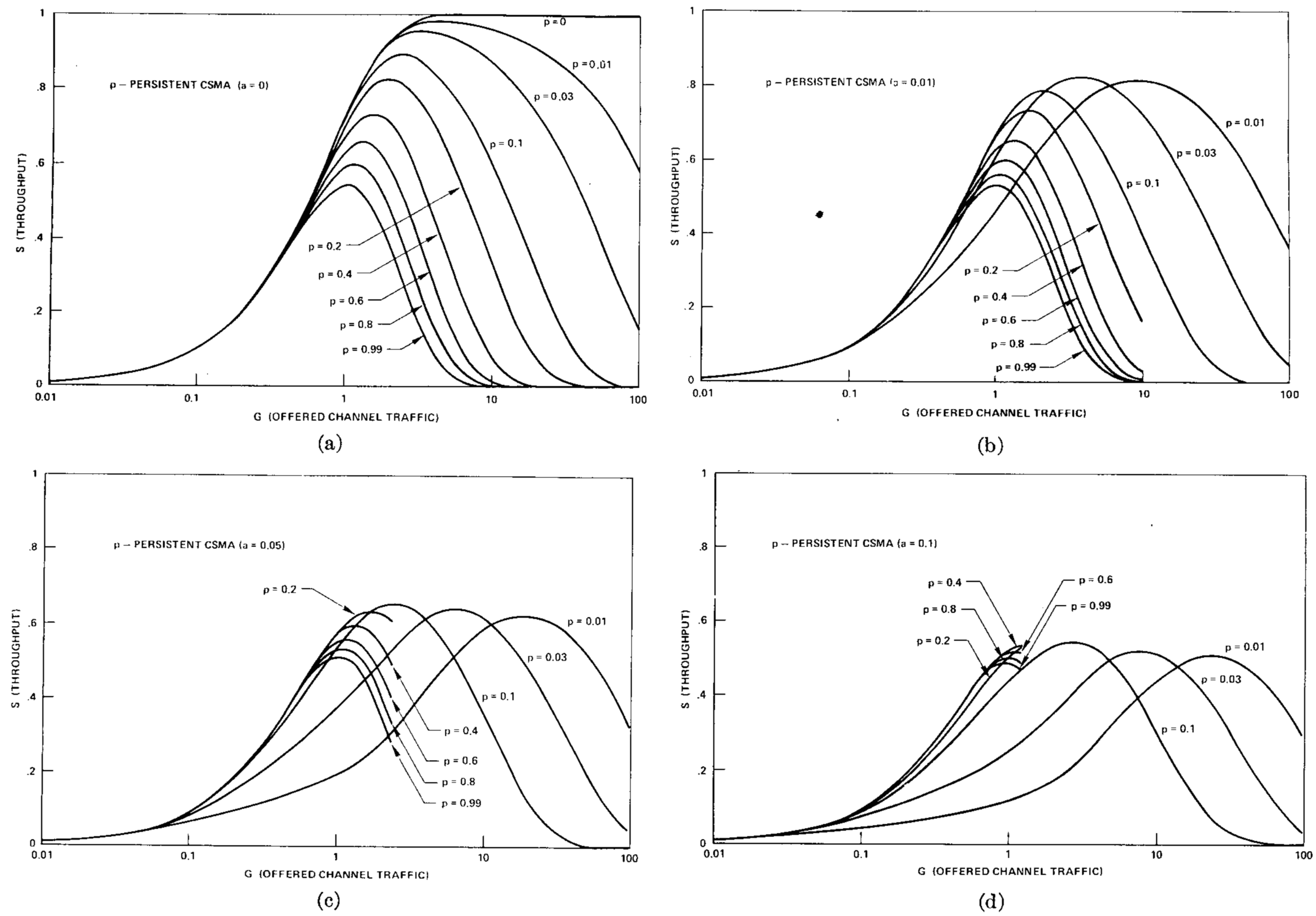

Fig. 7. Channel throughput in p-persistent CSMA. (a) $a = 0$. (b) $a = 0.01$. (c) $a = 0.05$. (d) $a = 0.1$.

and

$$P_s = \sum_{n=1}^{\infty} \frac{q^{n-1}G^n e^{-G}}{n!(1-e^{-G})} = \frac{qe^{-G}(e^{qG}-1)}{1-e^{-G}}.$$

In particular, $p \to 0$ gives $P_s(n) \to 1$, for all n, and $P_s \to 1$. In this limit the throughput is then given by

$$S(G,p \to 0, a = 0) \to \frac{G}{G + e^{-G}} \quad (52)$$

which shows that a channel capacity of 1 can be achieved when $G \to \infty$.

For each value of a, one can plot a family of curves S versus G with parameter p [as shown in Fig. 7 (a)–(d)]. The channel capacity for each value of p can be numerically determined at an optimum value of G. In Fig. 8 we show the channel capacity as a function of p, for $a = 0$, 0.01, 0.05, and 0.1. We note that the capacity is not very sensitive to small variations of p; for $a = 0.01$, it reaches its highest value (i.e., the channel capacity for this protocol) at a value $p = 0.03$. When $p = 1$, the (slotted) p-persistent CSMA reduces to the slotted 1-persistent CSMA. Indeed we can check that, when $p = 1$, (24) reduces to (19), since P_s, $\bar{l}$, and $\bar{l}'$ then become

$$P_s = \frac{aGe^{-G}}{1 - e^{-aG}}$$

$$\bar{l}' = \bar{l} = 0.$$

E. Performance Comparison and Sensitivity of Capacity to the Parameter a

To summarize, we plot in Fig. 9 for $a = 0.01$, S versus G for the various access modes introduced so far and thus show the relative performance of each, as also indicated in Table I.

While the capacity of ALOHA channels does not depend on the propagation delay, the capacity of a CSMA channel does. An increase in a increases the vulnerable period of a packet. This also results in "older" channel state information from sensing. In Fig. 10 we plot, versus a, the channel capacity for all of the above random-access modes. We note that the capacities for nonpersistent and p-persistent CSMA are more sensitive to increases in a, as compared to the 1-persistent scheme. Nonpersistent CSMA drops below 1-persistent for larger a. Also, for large a,

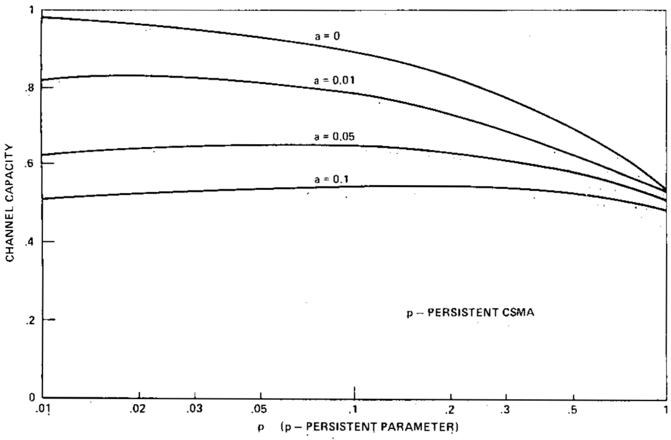

Fig. 8. p-persistent CSMA: effect of p on channel capacity.

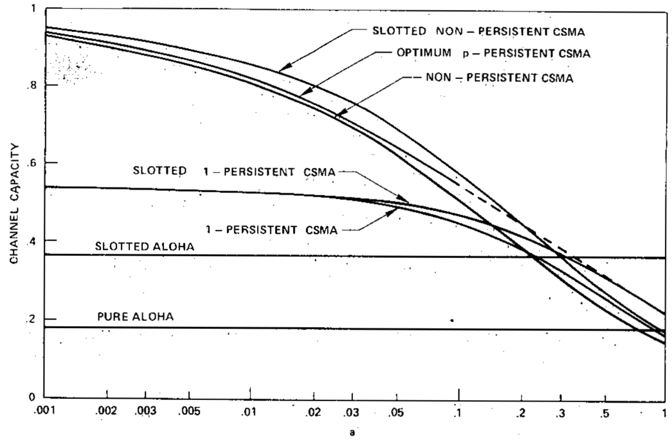

Fig. 10. Effect of propagation delay on channel capacity.

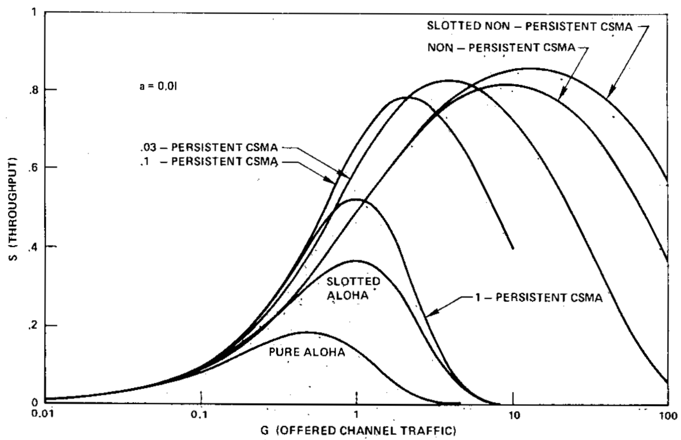

Fig. 9. Throughput for the various access modes ($a = 0.01$).

TABLE I
CAPACITY C FOR THE VARIOUS PROTOCOLS CONSIDERED ($a = 0.01$)

Protocol	Capacity C
Pure ALOHA	0.184
Slotted ALOHA	0.368
1-Persistent CSMA	0.529
Slotted 1-Persistent CSMA	0.531
0.1-Persistent CSMA	0.791
Nonpersistent CSMA	0.815
0.03-Persistent CSMA	0.827
Slotted Nonpersistent CSMA	0.857
Perfect Scheduling	1.000

slotted ALOHA (and even "pure" ALOHA) is superior to any CSMA mode since decisions based on partially obsolete data are deleterious; this effect is due in part to our assumption about the constant propagation delay. (For p-persistent, numerical results are shown only for $a \leq 0.1$. Clearly, for larger a, optimum p-persistent is lower-bounded by 1-persistent.)

V. DELAY CONSIDERATIONS

A. *Delay Model*

In the previous section, we analyzed the performance of CSMA modes in terms of maximum achievable throughput. We now introduce the expected packet delay D defined as the average time from when a packet is generated until it is successfully received.

Our principal concern in this section is to investigate the tradeoff between the average delay D and the throughput S.

As we have already stated, for the correct operation of the system, a positive acknowledgment scheme is needed. If an acknowledgment is not received by the sender of a packet within a specified time-out period, then the packet is retransmitted (incurring the random retransmission delay X, introduced to avoid repeated conflicts). For the present study, we assume the following.

Assumption 3: The acknowledgment packets are always correctly received with probability one.

The simplest way to accomplish this is to create a separate channel[11] (assumed to be available) to handle acknowledgment traffic. If sufficient bandwidth is provided to this channel overlaps between acknowledgment packets are avoided, since a positive acknowledgment packet is created only when a packet is correctly received, and there will be at most one such packet at any given time. Thus, if T_a denotes the transmission time of the acknowledgment packet on the separate channel, then the time-out for receiving a positive acknowledgment is $T + \tau + T_a + \tau$, provided that we make the following assumption.

Assumption 4: The processing time needed to perform the sumcheck and to generate the acknowledgment packet is negligible.

Assumption 2 further simplifies our delay model by implicitly assuming that the probability of a packet's success is the same whether the packet is new or has been blocked, or interfered with any number of times before; this probability is simply given by the throughput equation, i.e.,

$$P_s = \frac{S}{G} = \frac{\text{throughput}}{\text{offered traffic}}.$$

Bearing these assumptions in mind, we can write the delay equations for each of the previous access modes.

[11] The reader is referred to [16] for a study of the effect of acknowledgment traffic on channel throughput when acknowledgment packets are carried by the same channel.

As an example let us consider the ALOHA mode. Let R be the average delay between two consecutive transmissions (i.e., a retransmission) of a given packet. R consists of the transmission time of the packet, the transmission time of the acknowledgment packet, the round-trip propagation delay, and the average retransmission delay, that is

$$R = T + \tau + T_a + \tau + \bar{X}.$$

Using our normalized time units, we have

$$R = 1 + 2a + \alpha + \delta \tag{53}$$

where $\alpha = T_a/T$. Since $(G/S - 1)$ is the average number of retransmissions required, the average delay is given by

$$D = \left(\frac{G}{S} - 1\right)R + 1 + a. \tag{54}$$

(Special attention must be devoted to the CSMA modes in which packets may incur pretransmission delays, and in which all arrivals do not necessarily correspond to actual transmissions. The delay equations and their derivations are given in Appendix B.)

Let us begin with some comments concerning the above delay equations. First, G/S as obtained from the throughput equations rests on two important and strong Assumptions 1 and 2; namely, that we have an independent Poisson point process and that δ is infinite, or large compared to the transmission time (in which case delays are also large and unacceptable). On the other hand, δ cannot be arbitrarily small. It is intuitively clear that when a certain backlog of packets is present, the smaller δ is, the higher is the level of interference and hence the larger is the offered channel traffic G. Thus, $G = G(S,\delta)$ is a decreasing function of δ such that the average number of transmissions per packet, $[G(S,\delta)]/S$, decreases with increasing values of δ, and reaches the asymptotic value predicted by the throughput equation. Thus, for each S, a minimum delay can be achieved by choosing an optimal δ. Such an optimization problem is difficult to solve analytically, and simulation techniques have been employed in our evaluations below.[12]

Before we proceed with the discussion of simulation results, we compare the various access modes in terms simply of the average number of transmissions (or average number of schedulings[13]) G/S. For this purpose, we plot G/S versus S in Fig. 11 for the ALOHA and CSMA modes, when $a = 0.01$. Note that CSMA modes are superior in that they provide lower values for G/S than the ALOHA modes. Furthermore, for each value of the throughput, there exists a value of p such that p-persistent is optimal. For small values of S, $p = 1$ (i.e., 1-persistent) is optimal. As S increases, the optimum p decreases.

[12] We have been able to solve the problem analytically in the case of the nonpersistent CSMA when we are in presence of a large population but with a finite number of users; all conclusions obtained from simulation in Section V-B have been verified by the analysis. For this the reader is referred to reference [16].

[13] For the nonpersistent and p-persistent CSMA, G measures the offered channel traffic and not the actual channel traffic. G/S represents, then, the average number of times a packet was scheduled for transmission before success.

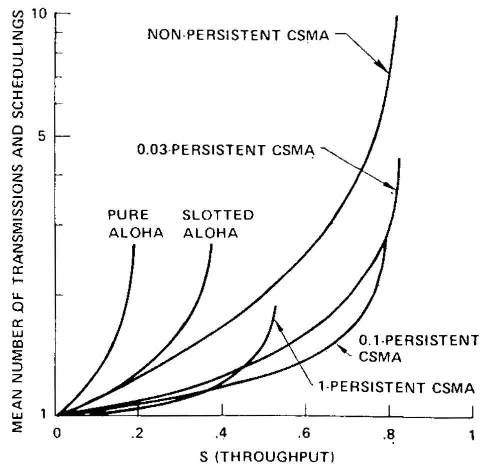

Fig. 11. G/S versus throughput ($a = 0.01$).

B. Simulation Results

The simulation model is based on all system assumptions presented in Section II. However, we relax Assumptions 1 and 2 concerning the retransmission delay and the independence of arrivals for the offered channel traffic. That is, in the simulation model, only the newly generated packets are derived independently from a Poisson distribution; collisions and uniformly distributed random retransmissions are accounted for without further assumptions.

In general, our simulation results indicate the following.

1) For each value of the input rate S, there is a minimum value δ for the average retransmission delay variable, such that below that value it is impossible to achieve a throughput equal to the input rate.[14] The higher S is, the larger δ must be to prevent a constantly increasing backlog, i.e., to prevent the channel from saturating. In other words, the maximum achievable throughput (under assumed stable conditions) is a function of δ, and the larger δ is, the higher is the maximum throughput.

2) Recall that the throughput equations were based on the assumption that $\bar{X}/T = \delta \gg 1$. Simulation shows that for finite values of δ, $\delta > \delta_0$, but not too large compared to 1, the system already "reaches" the asymptotic results ($\delta \to \infty$). That is, for some finite values of δ, Assumption 2 is excellent and delays are acceptable. Moreover, the comparison of the (S,G) relationship as obtained from simulation and the results obtained from the analytic model exhibits an excellent match. Simulation experiments were also conducted to find the optimal delay; that is, the value of $\delta(S)$ which allows one to achieve the indicated throughput with the minimum delay.

Finally, in Fig. 12[15] we give the throughput-minimum delay tradeoff for the ALOHA and CSMA modes (when $a = 0.01$). *This is the basic performance curve.* We conclude

[14] Such behavior is characteristic of random multiple-access modes. Similar results were already encountered by Kleinrock and Lam [13] when studying slotted ALOHA in the context of a satellite channel.

[15] In Fig. 12, the curve corresponding to slotted ALOHA is obtained from the analytical model developed in [13] successfully verified by simulation.

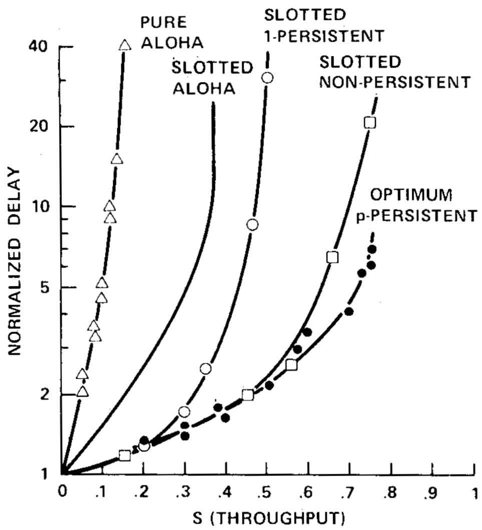

Fig. 12. Throughput-delay tradeoffs from simulation ($a = 0.01$).

that the optimum p-persistent CSMA provides us with the best performance; on the other hand the performance of the (simple) nonpersistent CSMA is quite comparable.

VI. SUMMARY AND DISCUSSION

We have introduced and evaluated the new CSMA mode and have shown it to be an efficient means for randomly accessing packet switched radio channels which have a small ratio of propagation delay to packet transmission time. Just as with most "contention" systems, these random multi-access broadcast channels (ALOHA, CSMA) are characterized by the fact that the throughput goes to zero for large values of channel traffic. At an optimum traffic level, we achieve a maximum throughput which we define to be the system capacity. This and the throughput-delay performance were obtained by a steady-state analysis under the assumption of equilibrium conditions.

However, these channels exhibit unstable behavior at most input loads as shown by Kleinrock and Lam [18]. In this last reference, the dynamic behavior and stability of an ALOHA channel are considered; quantitative estimates for the relative stability of the channel are given, which indicate the need for special control procedures to avoid a collapse. Optimal control procedures have been found [14], [15] and similar procedures are necessary for CSMA as well, since it can be shown [16] that CSMA exhibits similar unstable behavior.

Throughout the paper, it was assumed that all terminals are within range and in line-of-sight of each other. A common situation consists of a population of terminals, all within range and communicating with a single "station" (computer center, gate to a network, etc.) in line-of-sight of all terminals. Each terminal, however, may *not* be able to hear all the other terminals' traffic. This gives rise to what is called the "*hidden-terminals*" problem. The latter badly degrades the performance of CSMA as shown in Part II of this paper [19]. Fortunately, in a single-station environment, the hidden-terminal problem can be eliminated by dividing the available bandwidth into two separate channels: a busy tone channel and a message channel. As long as the station is receiving a signal on the message channel, it transmits a busy tone signal on the busy tone channel (which terminals sense for channel state information). The CSMA with a busy tone under a nonpersistent protocol has been analyzed. It is shown to provide a maximum channel capacity of approximately 0.65 when $a = 0.01$ for a channel bandwidth W of 100 kHz (modulated at 1 bit/Hz); when $W = 1$ MHz and $a = 0.01$, the channel capacity is 0.71 [19]. These values compare favorably with the capacity of 0.815 for nonpersistent CSMA with no hidden terminals.

APPENDIX A

SMALL p APPROXIMATIONS IN p-PERSISTENT CSMA

We claim, for small p, that $S(G,p,a)$ may be approximated by

$$S'(G,p,a) = \frac{(1-e^{-aG})[\hat{P}_s'\pi_0 + \hat{P}_s(1-\pi_0)]}{(1-e^{-aG})[a\hat{\bar{t}}'\pi_0 + a\hat{\bar{t}}(1-\pi_0) + 1 + a] + a\pi_0} \qquad \text{(A1)}$$

where $\hat{P}_s'$, $\hat{P}_s$, $\hat{\bar{t}}$, and $\hat{\bar{t}}'$ are defined hereafter in the proof.

Proof: We show here that, with some approximations, we can get a closed-form solution for the throughput when p has *small values* ($p < 0.1$). These approximations are validated by comparing the results obtained in this section with those obtained from Section IV-D for $p = 0.1$.

For the distribution of idle time between two TP's, we have from (26)

$$\Pr\{t_n > k\} = q^{(k+1)n} \exp\left\{g\left(\frac{q(1-q^k)}{p} - k\right)\right\}. \qquad \text{(A2)}$$

When p is small, we may make the following approximation (actually a lower bound):

$$q^k = (1-p)^k \simeq 1 - kp \qquad \text{(A3)}$$

and therefore we may rewrite (A2) as

$$\Pr\{t_n > k\} \simeq q^{(k+1)n}e^{-kpg} = q^n[q^n e^{-pg}]^k. \qquad \text{(A4)}$$

Let $t_{n>}^*(z)$ and $t_n^*(z)$ be the generating functions defined by

$$t_{n>}^*(z) \triangleq \sum_{k=0}^{\infty} \Pr\{t_n > k\} z^k \qquad \text{(A5)}$$

$$t_n^*(z) \triangleq \sum_{k=0}^{\infty} \Pr\{t_n = k\} z^k. \qquad \text{(A6)}$$

We have

$$t_{n>}^*(z) = q^n \sum_{k=0}^{\infty} (q^n e^{-pg} z)^k = \frac{q^n}{1 - q^n e^{-pg} z}. \qquad \text{(A7)}$$

Since

$$\Pr\{t_n = k\} = \Pr\{t_n > k-1\} - \Pr\{t_n > k\}, \qquad k > 0$$

and

$$\Pr\{t_n = 0\} = 1 - \Pr\{t_n > 0\},$$

we have

$$t_n^*(z) = 1 + (z-1)t_{n>}^*(z) = 1 + \frac{q^n(z-1)}{1 - q^n e^{-pg}z}. \quad \text{(A8)}$$

The averages defined in (29) can now be written as

$$\bar{t}_n = \left.\frac{\partial t_n^*(z)}{\partial z}\right|_{z=1} = \frac{q^n}{1 - q^n e^{-pg}}. \quad \text{(A9)}$$

Equation (30), which defines $\bar{t}$ as $\sum_{n=1}^{\infty} \bar{t}_n \pi_n/(1-\pi_0)$, does not lead to a closed-form expression. Instead, we replace $\bar{t}$ by $\hat{\bar{t}}$, which is defined as

$$\hat{\bar{t}} = \frac{C}{1 - Ce^{-pg}} \quad \text{(A10)}$$

where $C = \sum_{n=1}^{\infty} q^n \pi_n/(1-\pi_0)$. ($\hat{\bar{t}}$ is smaller than $\bar{t}$ since $\bar{t}_n = q^n/(1-q^n e^{-pg})$ is a convex function of q^n.)

C can be expressed as

$$C = \frac{\exp\{-(1+a)pG\} - \pi_0}{1-\pi_0} = \frac{\pi_0^p - \pi_0}{1-\pi_0} \quad \text{(A11)}$$

and therefore,

$$\hat{\bar{t}} = \frac{\pi_0^p - \pi_0}{1 - \pi_0 - (\pi_0^p - \pi_0)e^{-pg}}. \quad \text{(A12)}$$

To find the probability of success over TP $\mathfrak{I}_i$, $i \neq 1$, we first define the following generating functions:

$$L_n^*(z) \triangleq \sum_{l=n-1}^{\infty} \Pr\{L_n = l\} z^l \quad \text{(A13)}$$

$$L_n^*(z/k) \triangleq \sum_{l=n-1}^{\infty} \Pr\{L_n = l/t_n = k\} z^l. \quad \text{(A14)}$$

It is clear that

$$L_n^*(z/k) = \exp\{kg(z-1)\} z^{n-1}. \quad \text{(A15)}$$

Removing the condition on k, we get

$$\begin{aligned} L_n^*(z) &= \sum_{k=0}^{\infty} L_n^*(z/k) \cdot \Pr\{t_n = k\} \\ &= z^{n-1} \sum_{k=0}^{\infty} \exp\{kg(z-1)\} \cdot \Pr\{t_n = k\} \\ &= z^{n-1} t_n^*(\exp\{g(z-1)\}) \\ &= \frac{q^n(\exp\{g(z-1)\} - 1)z^{n-1}}{1 - q^n \exp\{-pg\}\exp\{g(z-1)\}} + z^{n-1}. \end{aligned} \quad \text{(A16)}$$

The probability of success $P_s(n)$, defined in (33), is now simply expressed (since $1 - q^l \approx lp$) as

$$P_s(n) = L_n^*(q) = q^{n-1} - \frac{(1-e^{-gp})q^{2n-1}}{1 - q^n e^{-2gp}}. \quad \text{(A17)}$$

Here again, (34) defines

$$P_s = \sum_{n=1}^{\infty} P_s(n) \cdot \frac{\pi_n}{1-\pi_0}$$

which does not lead to a closed-form expression. Instead, we replace P_s by $\hat{P}_s$, which is defined as

$$\hat{P}_s = \frac{C}{q} - \frac{(1-e^{-gp})C'}{q(1 - Ce^{-gp})} \quad \text{(A18)}$$

where C is as expressed in (A11), and

$$C' = \sum_{n=1}^{\infty} q^{2n} \frac{\pi_n}{1-\pi_0} = \frac{\pi_0^{1-q^2} - \pi_0}{1-\pi_0}. \quad \text{(A19)}$$

Finally, $\hat{P}_s$ can be expressed as shown in the following equation:

$$\hat{P}_s = \frac{\pi_0^p - \pi_0}{q(1-\pi_0)} - \frac{(1 - \exp\{-gp\})(\pi_0^{1-q^2} - \pi_0)}{q(1-\pi_0) - q\exp\{-2gp\}(\pi_0^p - \pi_0)}. \quad \text{(A20)}$$

The quantities $\hat{\bar{t}}'$ and $\hat{P}_s'$ are readily obtained from (A12), and (A20), respectively, by replacing

$$\pi_0 \triangleq \exp\{-G(1+a)\}$$

by the quantity e^{-g}. The substitution of $\hat{P}_s$, $\hat{\bar{t}}$, $\hat{P}_s'$, and $\hat{\bar{t}}'$ for P_s, $\bar{t}$, P_s', and $\bar{t}'$, respectively, in (46) provides us with a closed-form solution for $S(G,p,a)$ when p is small.

In Table II, we compare for $p = 0.1$ the "exact" results obtained from Section IV-D to those obtained by the approximation; note that the closed-form solution is quite satisfactory for $p < 0.1$.

APPENDIX B

DELAY EQUATIONS

A. Nonpersistent CSMA

In this case, the average delay R between two successive sense points of the same packet is

$$R = \begin{cases} 1 + \alpha + 2a + \delta, & \text{if the packet is transmitted} \\ \delta, & \text{if the packet is blocked.} \end{cases} \quad \text{(B1)}$$

Let P_b be the probability that an arrival gets blocked (i.e., senses the channel busy). We have

$$1 - P_b = \frac{a + 1/G}{\bar{C}} = \frac{1 + aG}{1 + G(1 + a + \bar{Y})}. \quad \text{(B2)}$$

Under the traffic independence assumption, the rate of

TABLE II
COMPARISON OF RESULTS FOR THROUGHPUT S OBTAINED FROM THE EXACT ANALYSIS (24) AND RESULTS OBTAINED FROM THE APPROXIMATION (APPENDIX A) WHEN $p = 0.1$

	$a = 0.01$		$a = 0.05$	
G	Exact	Approximate	Exact	Approximate
0.1	0.098	0.098	0.095	0.094
0.2	0.192	0.192	0.179	0.178
0.3	0.279	0.279	0.252	0.251
0.4	0.358	0.358	0.316	0.314
0.5	0.428	0.428	0.370	0.367
0.6	0.490	0.490	0.417	0.413
0.7	0.544	0.544	0.457	0.453
0.8	0.589	0.590	0.490	0.486
0.9	0.628	0.630	0.519	0.515
1.0	0.661	0.663	0.543	0.539
1.1	0.689	0.691	0.563	0.560
1.2	0.711	0.714	0.580	0.578
1.3	0.730	0.733	0.594	0.593
1.4	0.745	0.749	0.606	0.605
1.5	0.757	0.761	0.616	0.616
1.6	0.766	0.771	0.624	0.625
1.7	0.773	0.778	0.630	0.632
1.8	0.778	0.784	0.635	0.638
1.9	0.781	0.787	0.639	0.643
2.0	0.783	0.790	0.642	0.647
2.1	0.784	0.791	0.644	0.649
2.2	0.784	0.791	0.645	0.651
2.3	0.783	0.790	0.646	0.653

actual transmissions is given by

$$H = G(1 - P_b).$$

Since $(H/S) - 1$ represents the average number of actual retransmissions per packet, the average delay D is therefore

$$D = (H/S - 1)[1 + \alpha + 2a + \delta][(G - H)/S]\delta + 1 + a \quad \text{(B3)}$$

where G/S is given by the nonpersistent CSMA throughput equation (3).

If we choose to treat all packet arrivals in a uniform manner, we may assume that when a packet is blocked, it behaves as if it could transmit, and learned about its blocking only T_a seconds after the end of its "virtual" transmission. With this simplification, the delay equation is

$$D = (G/S - 1)(1 + 2a + \alpha + \delta) + 1 + a \quad \text{(B4)}$$

thus introducing an additional delay equal to $(GP_b/S)[1 + \alpha + 2a]$.

B. 1-Persistent CSMA

Unlike the ALOHA channel, a packet on a CSMA channel incurs an additional pretransmission delay r, if upon its arrival, that packet detects the channel busy. Recall that the probability of finding the channel busy is given by (see Section IV-C)

$$\Pr\{\text{a packet finds the channel busy}\} = \frac{\bar{B} - a/q_0}{\bar{B} + \bar{I}} = \frac{1 + \bar{Y}}{q_0(\bar{B} + \bar{I})} \quad \text{(B5)}$$

where $\bar{B}$, $\bar{I}$, $\bar{Y}$, and q_0 are given in (16), (12), (7), and (15), respectively.

Under the condition that the packet found the channel busy, the average waiting time until the channel is detected idle (i.e., until the end of the TP) is simply equal [17] to $\overline{Z^2}/2\bar{Z}$ by the Poisson assumption. The second moment of Z is simply given by

$$\overline{Z^2} = \overline{(1 + Y)^2} = 1 + 2\bar{Y} + \overline{Y^2}.$$

From the distribution of Y given in (6) we then have

$$\overline{Z^2} = 1 + a^2 + 2(1 - 1/G)\bar{Y}. \quad \text{(B6)}$$

Therefore the average pretransmission delay $\bar{r}_1$ can be easily expressed as

$$\bar{r}_1 = \frac{1 + a^2 + 2(1 - 1/G)\bar{Y}}{2(1 + \bar{Y})} \cdot \Pr\{\text{the packet finds the channel busy}\} = \frac{1 + a^2 + 2(1 - 1/G)\bar{Y}}{2q_0(\bar{B} + \bar{I})}. \quad \text{(B7)}$$

Finally, the expected packet delay is

$$D = (G/S - 1)(1 + 2a + \alpha + \delta + \bar{r}_1) + \bar{r}_1 + 1 + a \quad \text{(B8)}$$

where G/S is given by the 1-persistent CSMA throughput equation (10).

C. p-Persistent CSMA

Similar to the special case of 1-persistent CSMA, a packet in this general scheme incurs an initial delay which we denote by r_p. In order to compute its expected value $\bar{r}_p$, one must consider the following situations.

1) An arbitrary packet, upon arrival, will find the channel idle with probability $\bar{I}/(\bar{B} + \bar{I})$, in which case its average initial wait is $a\bar{l}'$.

2) An arbitrary packet, upon arrival, will find the channel in the first IRTD (first t' seconds) of a busy period with probability $a\bar{l}'/(\bar{B} + \bar{I})$. In this case, its average initial delay is $a\overline{l'^2}/2\bar{l}'$.

3) An arbitrary packet, upon arrival, will find the channel in the remaining part of a busy period with probability $(B - a\bar{l}')/(\bar{B} + \bar{I})$, in which case the average initial wait is $\overline{(1 + a + at)^2}/2(1 + a + a\bar{l})$.

Therefore

$$\bar{r}_p = \frac{\bar{I}}{\bar{B} + \bar{I}} a\bar{l}' + \frac{a\bar{l}'}{\bar{B} + \bar{I}} \cdot \frac{a\overline{l'^2}}{2\bar{l}'} + \frac{B - a\bar{l}'}{\bar{B} + \bar{I}} \cdot \frac{\overline{(1 + a + at)^2}}{2(1 + a + a\bar{l})}. \quad \text{(B9)}$$

Treating all transmissions and schedulings uniformly (by introducing artificial delays due to "virtual" transmissions and acknowledgment), the expected delay can simply be expressed as

$$D = (G/S - 1)[1 + 2a + \delta + \bar{r}_p] + 1 + a + \bar{r}_p \quad \text{(B10)}$$

where G/S is given by the p-persistent CSMA throughput equation (24).

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